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Surname:

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Prep School:

Malvern College Academic
Scholarship Specimen Examinations

13+ MATHS PAPER 2

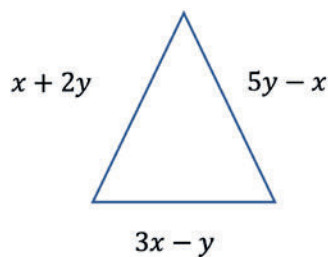
- » This examination is 60 minutes long.
- » This examination has 75 marks in total.
- » No Calculators are allowed for this paper
- » Section A (25 marks): Multiple choice but with working and explanations required.
- » Section B (50 marks): Investigative questions. These are more challenging and require full solutions.

SECTION A (25 MARKS):

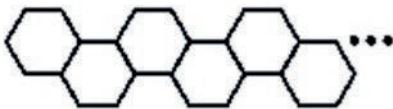
Multiple Choice (2 marks will be awarded for a correct answer and up to 3 marks for good explanations)

Answer in Booklet provided

1. Which of the following is the longest period of time?
A 3002 hours **B** 125 days **C** 17.5 weeks **D** 4 months **E** 1/3 of a year
2. In the equilateral triangle shown, the lengths of the sides are given. Which of the following could NOT be the values of x and y ?



- A** (18,12) **B** (15,10) **C** (12,8) **D** (10,6) **E** (3,2)
3. Sam's 101st birthday is tomorrow, so Sam's age in years changes from a square number (100) to a prime number (101). How many times has this happened in Sam's lifetime?
A 1 **B** 2 **C** 3 **D** 4 **E** 5
4. Nicky has to choose 7 different positive whole numbers whose mean is 7. What is the largest possible such number she can choose?
A 7 **B** 28 **C** 34 **D** 43 **E** 49
5. A shape consisting of a number of regular hexagons is made by continuing to the right the pattern shown in the diagram, with each extra hexagon sharing one side with the preceding one. Each hexagon has a side length of 1 cm.



How many hexagons are required for the perimeter of the shape to have length 2022 cm.

- A** 335 **B** 405 **C** 505 **D** 672 **E** 1011

SECTION B (50 MARKS):

Each question is worth 10 marks. Greater marks are given for clear detailed working and explanation. You should not expect to finish all of these but attempt as many as you can.

Answer on lined paper

6. Find two consecutive integers such that the sum of their squares is 144 less than the square of their sum.

(Hint: consecutive numbers are x and $x+1$, no marks will be awarded for a trial and improvement method)

7. Show that $(x+1)^3+(x-1)^3+(-x)^3+(-x)^3 \equiv 6x$

Hence

- a. Express 12 as the sum of 4 cubes, some of which may be negative numbers.
- b. Express 17 as the sum of 5 cubes, some of which may be negative.
8. A Rubik's cube is an example of a 3×3 cube made up of twenty-seven 1×1 cubes. Arthur takes a Rubik's cube and paints every visible face red. He then dismantles the cube into its constituent 1×1 cubes. Eight of these cubes have red paint on three of their faces and one of them has no paint at all.
- a. Find how many of the smaller 1×1 cubes have:
- One face which has been painted red
 - Two faces which have been painted red

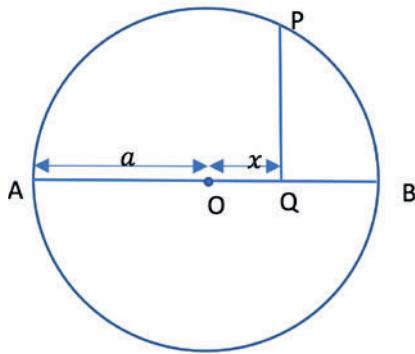
He now takes a larger 4×4 cube made in a similar way of 1×1 cubes and does the same thing.

- b. Find how many of the smaller 1×1 cubes have
- Three faces painted red
 - No faces painted red
 - One faces painted red
 - Two faces painted red.

Arthur now takes a larger $n \times n$ cube and repeats the process.

- c. Find in terms of n , how many of the smaller cubes have
- Three faces painted red
 - No faces painted red
 - One faces painted red
 - Two faces painted red.

9. In the diagram below, AOB is the diameter of a circle, centre O and radius a, and Q is a point between O and B such that OQ=x. P lies on the circle and PQ is perpendicular to AB.



- a. Find an expression for PQ^2 in terms of a and x.
 - b. Using your result prove that
 - i. $AP^2 = 2a^2 + 2ax$
 - ii. $BP^2 = 2a^2 - 2ax$
 - c. By considering the value of $AP^2 + BP^2$ what does this tell you about APB
10. It is true that if a,b,c and d are consecutive integers with $0 < a < b < c < d$ then $\sqrt{(abcd+1)} = bc-1$

$$\text{Let } N = \sqrt{(abcd+1)} = bc-1$$

For example: for $a=3$, $\sqrt{(3 \times 4 \times 5 \times 6 + 1)} = \sqrt{361} = 19$ and $bc-1 = 4 \times 5 - 1 = 19$

- a. Evaluate the following
 - i. Given that $a=8$, find N
 - ii. Given $N=109$, find a
 - iii. Given $d=32$, find N
 - iv. Given $ad=108$, find N
- b. Show algebraically that $\sqrt{(abcd+1)} = bc-1$, letting $a=x$

ANSWER BOOKLET

Circle the correct answer and submit working below.

Question 1

A B C D E

Question 2

A B C D E

Question 3

A B C D E

Question 4

A B C D E

Question 5

A B C D E

END OF QUESTION PAPER